1851MNRAS..11..187D

of its having formerly been overlooked. It may even have been mistaken for a very narrow and unusually dark belt; or for a portion of one, the rest being concealed by the bright ring crossing the planet. The position of the ring subsequently to the beginning of last October, was especially favourable to its visibility; for it was projected on to the very bright region extending from the planet's centre towards its north pole, by contrast with which it would appear unusually dark."

On a Photometrical Method of Determining the Magnitudes of Telescopic Stars. By the Rev. W. R. Dawes.

"The magnitudes of telescopic stars are so variously assigned by different observers, as to render it impossible correctly to anticipate the appearance of such an object in a telescope of any particular kind and dimensions, without having reference to the scale of magnitudes adopted by the observer who has assigned the magnitude of the star in question; and even then it is necessary to suppose that his habit of estimating is fixed and tolerably uniform. The differences among observers of great experience and celebrity are much greater than would probably be imagined by those who have not been led to examine the subject, and clearly show that widely different *scales* of magnitude have been adopted; and though, provided each observer's habit of estimation is generally consistent with his own scale, it might not be very difficult to convert the magnitudes of one into those of another, yet the trouble of this is to be deprecated; and the different numerical denominations of the magnitude of the same star create complication and confusion where simplicity and uniformity are especially desirable.

"In examining the estimated magnitudes by different observers, one remarkable fact presents itself; namely, that there is a tendency common to almost all to underrate the magnitudes of stars when seen through a telescope. This is most strikingly exemplified by observations of stars visible with the naked eye, and whose magnitudes may therefore be estimated without telescopic aid. The following instances will afford illustrations of this:—

"ζ *Cancri.* This triple star is obvious to the naked eye, and is designated by Argelander, in his *Uranometria Nova*, as of magnitude 5·4 (or about 4·7). Telescopically its three components have been estimated thus: by Sir James South (S), 7, 8, 8; by Sir John Herschel (H), 8, 8½, 8½; by Struve (Σ), 5·0, 5·7, 5·5: the latter therefore agreeing with Argelander (A).

"32 *Orionis.* Obvious to the naked eye, and designated by A as 5·6 (or about 5·3); by H, 7½ and 8½ (mean of two estimations); by Σ, 5·2 and 6·7, nearly agreeing with A.

"ζ *Aquarii.* A double star whose components are nearly equal, appearing to the naked eye as a bright 4 mag., and designated by

1851MNRAS..11..187D

A as 3·4 (or about 3·3); by S, 7 and 7¼; by Σ, 4·0 and 4·1, agreeing therefore very nearly with A.

"δ *Serpentis.* A double star, to the naked eye a rather small 3 mag., and by A called 3·4 (= about 3·3). The components are called by Σ 3 and 4, making the star about half a magnitude brighter than A's estimation; while by S they are estimated on *five different nights* as 8 and 9,—being a difference of five magnitudes.

"γ *Virginis.* A well-known double star, whose components are equal; to the naked eye a bright third mag., or, according to A., 3·2 (= about 2·7). By S, estimated on three nights as 8 and 8½, and on one night as 8½ and 9,—being a difference of more than five magnitudes.

"α *Cancri*, by A's naked-eye estimation, is of mag. 4; by Bessel (B) it is telescopically estimated as of the 6th mag.

"κ *Cancri*, estimated with the naked eye by A as of the 5th; telescopically by B, as of the 7th.

"ν *Leonis*, by A's naked-eye estimation, = 5; by B telescopically, = 7.

"The above are a few of the many examples which might be cited from the works of some of the most practised observers. That such stars as δ *Serpentis* and γ *Virginis*, which would by the naked eye be instantly missed from the constellations to which they belong, should be deliberately and repeatedly designated by any practised observer as of magnitudes so small as to be less visible than the planet *Neptune*, is certainly surprising, and strikingly shows the effect occasionally produced on these estimations by the use of a telescope, and especially under high magnifying powers, and perhaps an illuminated field. And it appears that even Bessel has not unfrequently been deceived into estimating stars plainly visible to the naked eye as of only the 7th magnitude.

"It must be acknowledged, however, that it is not very easy for an observer to maintain consistency even with his own scale of telescopic magnitudes, especially under the varying aspects produced by different magnifying powers. And even if it were otherwise, and consistency were usually or easily preserved, the adoption of a different scale by each observer, depending perhaps upon the optical power of the instrument he happens to employ, is productive of great confusion, and requires at any rate that the name of the observer should be affixed to the magnitudes which he assigns, or that the scale he uses should be distinctly stated, in order that some idea may be conveyed of the kind of object we may expect to meet with under the given designation.

"I am not aware upon what principles the scales of telescopic magnitude employed by some of the observers of the most extensive catalogues have been formed. It is evident that Lalande, Piazzi, Bessel, and Argelander, have not referred their estimations to precisely the same standard; the two last, however, differing but slightly, as might be expected from their early association in the Königsberg zone observations. But the adoption of a particular

1851MNRAS..11..187D

numerical magnitude to represent a certain degree of brightness has been, as far as I know, purely arbitrary. Even Struve, as we are informed by himself, was led to fix upon the twelfth magnitude to represent the smallest star usually visible in the Dorpat refractor, simply because the number of telescopic magnitudes would thus be the same as those customarily applied to the stars visible with the naked eye. (See Introduction to the *Mensuræ Micrometricæ*, p. lxvii.) Sir J. Herschel has carried his estimation of stars visible in his 20-foot front-view reflector down to the 20th magnitude; and Mr. Bond has done the same in respect to the Munich refractor having an aperture of 15 inches. Here also the limit of visibility appears to have been arbitrarily assigned. Sir J. Herschel has, however, given examples of the different units of his scale from the sixth downwards, in his *Introduction to the First Series of Double Stars* discovered in sweeping with his 20-foot reflector. (See *Memoirs of the Astronomical Society*, vol. ii. p. 464.) And in the Introduction to his Third Series, contained in vol. iii. p. 182, of the *Memoirs*, he says,—'The principle on which I have endeavoured to proceed, in estimating the relative magnitudes of stars below the sixth, is that of continually halving the light of each magnitude to give that of the next inferior denomination; so that, in fact, two stars of the 9th magnitude, so close together as not to be distinguished from one, shall affect the eye as a single star of the 8th,' &c.

" Professor Struve also informs us that he has adopted a somewhat similar plan in assigning the magnitudes of the two components of a double star; with this important variation, however, that a star having half the light of any given magnitude is considered to stand only *half* a magnitude lower in the scale; so that two stars of the 9th magnitude, when viewed with a power too low to separate them, shall affect the eye as one of magnitude $8\frac{1}{2}$; and if a star appearing to the naked eye as of the sixth magnitude were divided by the telescope into two equal stars, each of the components would be considered as of the magnitude $6\frac{1}{2}$; and so on. (See Introduction to the *Mensuræ Micrometricæ*, p. xlii., at the bottom.)

" Now it occurred to me, some years ago, that upon the same principle more completely and extensively carried out, the magnitudes of telescopic stars, or their quantity of light relatively to any given standard, might be pretty accurately determined by a photometrical process. It would certainly be convenient, for the sake of uniformity, that the ratio of light in any two given denominations of magnitude should be generally agreed upon, as the necessity for translating the language of one scale into that of another would be thereby avoided. In the case of the two scales just referred to, the Herschelian assigns a decrease of light according to the powers of $\frac{1}{2}$; while the Struvian assigns a decrease according to the powers of $\frac{1}{4}$; the units of numerical denomination increasing in arithmetical progression. The two scales therefore rapidly diverge, but are easily comparable; yet it would be more convenient if only one were employed.

1851MNRAS..11..187D

"The photometrical method which I would now propose is founded on the obvious principle that if there be two stars, A and B, of which B possesses *half* the brightness of A, then will B require *twice* the telescopic illuminating power to render its image equally bright with that of A; and the requisite increase of illuminating power is produced by employing *twice the area* of the object-glass, or object-mirror, which has been found to give to the image of A a definite degree of brightness. Thus the area employed must be inversely as the brightness of the object; and consequently the reciprocal of the area requisite to impart to the image of a star a definite degree of brightness becomes a measure of the intrinsic brightness, or magnitude, of that star.

"In order to the practical application of the proposed method, it is necessary previously to fix upon the magnitude from which to set out, and with which others are to be compared as a standard. To avoid as far as possible the uncertainty attending the denomination of *telescopic* magnitudes, I propose to employ as the standard an average of stars of the *sixth* magnitude, according to Argelander's *Uranometria Nova*. This may be taken as the limit of steady visibility by the average of *unassisted* vision on ordinary clear nights; and this *limit of steady visibility* is the *definite degree of brightness* to which all objects must be brought up in comparing their magnitudes with the standard magnitude.

"The next point to be determined is the *telescopic aperture* requisite for rendering average stars of the sixth magnitude just steadily visible. From theory it might readily be concluded, that, as *both* eyes are employed in unassisted vision, and as also much light is lost by reflection at the eight surfaces presented by the double object-glass and double eyepiece of an achromatic telescope, to which, moreover, only *one* eye is applied, it would require an achromatic aperture considerably *larger* than the pupil of the eye to meet the proposed conditions, especially when a magnifying power is employed which is large, compared with an aperture whose diameter is only two or three tenths of an inch. But experiment proves that in this case, as in many others, some circumstances, with which perhaps we are but imperfectly acquainted, greatly modify the deductions to which theory would lead. My experiments have been conducted principally with the finder of my Munich equatoreal, having an aperture of 1·53 inch, a focal length of 20 inches, and a magnifying power of 16½. Assuming that the pupil of my eye might probably be 0·2 of an inch in diameter, when stimulated by no more light than that of a clear moonless night, I employed an aperture of a quarter of an inch as making a fair allowance for the loss of light, the magnifying power, and the use of only one eye in observing. This aperture, however, proved to be much too large, stars of the sixth magnitude being far more than only just steadily visible with it. After various experiments, an aperture of 0·15 of an inch was fixed upon; and with this and the usual power of 16½, a list of 21 sixth-magnitude stars from Argelander's *Uranometria Nova* were examined. Of these, 9 ap-

1851MNRAS..11..187D

peared to be of precisely the standard brightness, being just steadily visible; 7 were rather too bright, and were estimated to be of magnitudes from one to five-tenths of a unit brighter than the sixth; while the remaining 5 were rather too faint, and were estimated at from 6·1 to 6¼. The mean of the whole gave $D = A - 0{\cdot}04$, or $6\,D = 6{\cdot}04\,A$. The average estimation may, therefore, be considered as identical.

" It is, however, obvious that the various degrees of sensibility of the optic nerve in different persons must render it necessary for each observer to determine experimentally for himself what aperture of the telescope he uses is requisite to give the desired visibility to average stars of the sixth magnitude; and from that to determine the apertures corresponding to the lower magnitudes. In all such cases the telescope and the observer's eye must be considered as constituting one compound optical instrument.

" It was at first my intention to adopt Sir J. Herschel's scale, as generally avoiding the use of half-magnitudes; but it immediately appeared that the magnitudes thus determined would be widely different from those adopted by the principal observers of extensive star-catalogues, especially Lalande, Piazzi, Bessel, and Argelander. I have, therefore, adopted Struve's ratio of progression, according to which, as will appear hereafter, the magnitudes assigned by the proposed method very nearly agree with the average estimations of the great observers just mentioned; and this I considered a sufficient reason for giving it the preference.

" Let, then, the numerical magnitudes of telescopic stars be assumed, so that one star of the sixth magnitude shall equal two stars of magnitude 6½; and one star of magnitude 6½ = two stars of magnitude 7, and so on. Then, putting m = the standard magnitude; a = the aperture necessary to show it; μ = any other inferior magnitude, expressed as usual by a larger numerical denomination; and α = the corresponding aperture necessary to render visible a star of the magnitude μ, we have $\alpha = a \,.\, 2^{\mu - m}$; by which formula the apertures in the following table, corresponding to the different magnitudes in the scale may be computed; 0·15 of an inch being the standard aperture corresponding to magnitude 6, *suited to my own eye.* I add also the corresponding apertures of reflecting telescopes of various constructions, the ratio of an equally illuminating aperture to the aperture of an achromatic refractor being assumed in the Herschelian, or front view, as 6 to 5; in the Newtonian, as 7 to 5; and in the Gregorian, as 8 to 5.

Σ's Magnit.	H's Magnit.	Achromat. Aperture.	Front View Aperture.	Newtonian Aperture.	Gregorian Aperture.
		inch.	inch.	inch.	inch.
6	6	0·15	0·18	0·21	0·24
6½	7	0·21	0·25	0·30	0·34
7	8	0·30	0·36	0·42	0·48
7½	9	0·42	0·51	0·60	0·68
8	10	0·60	0·72	0·85	0·96

1851MNRAS..11..187D

Σ's Magnit.	H's Magnit.	Achromat. Aperture. inch.	Front View Aperture. inch.	Newtonian Aperture. inch.	Gregorian. Aperture. inch.
8½	11	0·85	1·02	1·20	1·36
9	12	1·20	1·44	1·70	1·92
9½	13	1·70	2·04	2·40	2·72
10	14	2·40	2·88	3·39	3·84
10½	15	3·39	4·07	4·80	5·43
11	16	4·80	5·76	6·79	7·68
11½	17	6·79	8·15	9·60	10·86
12	18	9·60	11·52	13·58	15·36
12½	19	13·58	16·29	19·20	
13	20	19·20	23·04	27·15	
13½	21	27·15	32·58	38·40	
14	22	38·40	46·08	54·31	
14½	23	54·31	65·17	76·80	
15	24	76·80	92·16	108·61	

" In computing such a table, however, with any magnitude and corresponding aperture as the standard, the numbers may be more readily deduced from the standard aperture than by computing each by the formula. For the apertures suited to the succeeding units of magnitude below the sixth are deduced from the aperture suitable to the sixth, by continually multiplying by 2. The aperture adapted to the first half-magnitude below the sixth is obtained by multiplying the aperture for the sixth magnitude by the root of 2 (= 1·4142). And for the succeeding half-magnitudes the aperture thus found for magnitude 6½ is also continually multiplied by 2.

" As an example of the computation by the formula of the numbers expressing the achromatic apertures in the above table, let it be required to find the aperture (α) corresponding to the 12th magnitude in Struve's scale.

" Here $m = 6$; $a = 0·15$; $\mu = 12$; $\mu - m = 6$; and $\alpha = a \, . \, 2^{\mu - m} = 0·15 \times 2^{6} = 9·6$.

" It is very remarkable that the aperture thus determined to be requisite for rendering just visible to my own eye a star of Σ's 12th magnitude, should be precisely the aperture of the Dorpat refractor to which Struve has arbitrarily assigned that magnitude as the smallest visible on ordinary nights.

" Putting H = the numerical magnitude according to the Herschelian scale, and Σ = the numerical magnitude according to the Struvian scale, we have the formula, $H = 2\,\Sigma - 6$, by which the Struvian are converted into the Herschelian; and the formula, $\Sigma = \frac{H + 6}{2}$, by which the Herschelian are deducible from the Struvian.

" It must be acknowledged that the magnitudes actually

1851MNRAS..11..187D

assigned by Sir John Herschel to double stars discovered in his sweeps, frequently differ considerably from those to which they are entitled according to the comparative scale in the above table. Yet from several circumstances I am led to attribute this difference mainly to the great difficulty, amounting almost to impossibility, of assigning the proper place in the scale to objects differing so greatly in brightness as do the stars within the compass of the front view 20-foot reflector; and especially when caught hastily during sweeps, the principal objects of which are of quite another character. In several remarkable instances, however, where, from some particular interest attaching to the object, the magnitudes have been carefully estimated, the agreement of the estimation with the numbers in the table is striking. As examples, we may cite some of the objects specially mentioned by Sir John in his *Introduction to his Third Series*, as probably not within the reach of the Dorpat refractor. (*Mem. Ast. Society*, vol. iii. p. 182.) Of these, M. Struve selected three for examination, viz. κ *Geminorum*, ξ *Pegasi*, and α^2 *Cancri*. Of κ *Geminorum* the small companion is stated by H to be of magnitude 14; Σ assigns to it magnitude 10. ξ *Pegasi* is of magnitude 5 and 18, according to H; of 5 and 12, according to Σ. In these two instances the estimation of brightness is *identical;* the numerical difference of magnitude being precisely what the two scales require. To α^2 *Cancri* H assigns $4\frac{1}{2}$ and 20; Σ does not give his magnitudes; but having measured both the position and distance with the Dorpat telescope, the magnitude he would assign is certainly not below the 12th, perhaps about $11\frac{1}{2}$, which would agree with 17 H. Subsequently, the companions both of ξ *Pegasi* and α^2 *Cancri* were stated by H to be of 16 mag. (See *List of Test Objects* in *Mem. Ast. Society*, vol. viii. p. 31.) And this shows the difficulty of rightly designating these *intensiva* of faintness, without some photometrical standard, to which from time to time recourse may readily be had; for, according to the Herschelian scale, one star of the 16th magnitude is equal to 16 stars of the 20th. To these may be added the faint companion of τ *Boötis*, to which H assigns 16 mag. By my method it appears to be of magnitude $11\frac{1}{4}$, which corresponds to $16\frac{1}{2}$ H. In the comparative table, however, drawn up by Sir John Herschel from about 500 comparisons of the magnitudes assigned by himself in his sweeps, and by Struve, in his large catalogue of double stars (*Mem. Ast. Society*, vol. iii. p. 180), the *average* difference is found to be only about one magnitude. So far then, from the magnitudes of the smaller stars having been usually underrated by Sir John, the deviation from his own scale seems to have been decidedly in the opposite direction, and, as has been pointed out by himself, gradually increasing. (*Mem. Ast. Society*, vol. vi. p. 80.)

"Mr. Bond carries his numerical designation of the magnitudes of stars visible with the refractor of 15 inches in aperture, down to 20. His scale, therefore, appears so nearly to agree with the Herschelian that it may be deemed practically identical. For by the

1851MNRAS..11..187D

formula, $\alpha = a\,.\,2^{\mu - m}$, we find, that when $m = 6$, $a = 0{\cdot}15$, and $\alpha = 15$, the value of $\mu = 12{\cdot}64$, which is the magnitude according to Struve's scale; and this converted into the Herschelian scale $= 19{\cdot}28$; sufficiently near to 20 to require no further correction.

"But it is especially a matter of interest to ascertain how far the magnitudes, photometrically determined by the plan now proposed, agree with those which have been employed by the observers of the most extensive star-catalogues. To this end I have examined a considerable number of objects taken from Baily's edition of the *Hist. Cél.* of Lalande, from Weisse's Bessel, and from Argelander's zone observations. My selected working catalogues contained only the right ascension and declination of the objects, without any intimation of their magnitudes, and were arranged in the order of right ascension. The proper apertures, adapted to the case where $m = 6$ and $a = 0^{\text{in}}{\cdot}15$, having been cut out of card-board and laid out on a table in the observatory, the equatoreal was set to the place of the object; an estimation made of its magnitude from its appearance in the finder, in the first place without diminution of its aperture; and then such a diminishing aperture selected by trial as reduced the brightness of the object to the lowest point compatible with keeping it steadily in view when the eye was directed fully upon it. The designation of the star, with the aperture suitable to it, and the corresponding magnitude, were immediately entered in my journal; and it appeared that the finding of each object, determining its magnitude, and recording the particulars, occupied from three to four minutes: so that from fifteen to twenty could be thus determined in an hour. The nights devoted to this work were invariably those whose quality was not suitable for micrometrical operations, though apparently free from any considerable quantity of haze.

"In the following table, the first column, under the initial D, contains the magnitude, as determined by the proposed method; in the second column is stated the whole number of stars of each magnitude which have been used in the comparisons. The third column, under the initial B, contains the average corresponding magnitude attributed by Bessel to the stars whose number is given in the fourth column; the fifth and sixth columns contain the corresponding magnitudes and the number of stars from Argelander; and the seventh and eighth columns contain similar quantities from Lalande. The ninth column contains the differences between the magnitudes assigned by me and the mean of the corresponding magnitudes from Bessel, Argelander, and Lalande; and the tenth column contains the results of comparison of my magnitudes with the mean of Bessel and Argelander only. The stars from Lalande are all included among those taken from Bessel and Argelander. The whole number of stars compared is 187; of which 92 are from Bessel, and 95 from Argelander; and 81 are also found in Lalande.

1851MNRAS..11..187D

D.	Stars.	B.	Stars.	A.	Stars.	L.	Stars.	$D-\frac{B+A+L}{3}$	$D-\frac{B+A}{2}$
7·0	19	7·00	12	6·97	7	6·53	15	+0·17	+0·01
7·5	30	7·47	16	7·44	14	7·47	23	+0·04	+0·05
8·0	51	8·15	26	8·09	25	8·17	25	+0·14	−0·12
8·5	50	8·69	24	8·37	26	8·43	13	0·00	−0·03
9·0	37	8·99	14	8·86	23	8·70	5	+0·15	+0·08

"The only discrepancy of any consequence among the numbers in the table occurs in Lalande's estimations of stars which I have designated of the 7th magnitude. The difference amounts to nearly half a magnitude, and arises from the frequency with which Lalande has assigned the 6th magnitude to stars not contained in Argelander's *Uranometria*, and certainly not ordinarily visible to the naked eye. There is also a difference of 0·3 of a unit in the same column corresponding to my 9th magnitude; and the cause of this is found partly in Lalande's having more rarely used 9 as a designation of magnitude than either Bessel or Argelander has done; and partly, no doubt, in the small number of stars (only 5) of that magnitude found in Lalande among the stars compared.

"On the whole, the agreement of the magnitudes determined by the proposed method, with those employed by the three great zone observers, is far more close than I could have ventured to anticipate; and constitutes a strong additional recommendation of it on the ground of uniformity, so especially desirable in this department of practical astronomy.

"It has been supposed that the magnitudes assigned by Struve to the objects contained in his great Dorpat catalogue are usually much too high. Tested by my photometrical method, the average difference between his estimations and my own is rather more than one quarter of a magnitude, by which quantity his magnitudes are higher than mine. By comparison of 74 stars common to Struve and Bessel, between the sixth and ninth magnitudes, the mean difference results, $\Sigma = B - 0·40$. And by 32 telescopic stars common to Struve and Argelander the average difference comes out, $\Sigma = A - 0·46$; by which quantities, 0·40 and 0·46 respectively, Struve's magnitudes are higher than Bessel's and Argelander's. A comparison, however, of 80 stars in Argelander's *Uranometria*, from the 3d to the 6th magnitude, gives the mean difference in the estimations of those stars, $\Sigma = A - 0·005$ only. From the moderate number of stars compared in the above cases, the conclusions can be received only as showing the tendency; yet so great is the consistency and uniformity of estimation by these eminent observers, that I doubt if the results would be greatly modified by a more extensive comparison. And they are sufficient to show that the proposed method of photometrical determination gives results holding a middle place among those of the most careful and accurate observers, and differing from any of them on an average by only a small fraction of a unit of the scale adopted.

"One great recommendation of the method of apertures for the

1851MNRAS..11..187D

determination of magnitudes consists in the case with which it is employed; requiring no new instrument or eye-piece, or any variation in the arrangement of the telescope or its eye-pieces, which may be in use at the time. Nothing more is wanted than a few circular apertures neatly cut out of card-board, and of such a size as to be applied to the object-end of the finder, for the determination of magnitudes, as far as that instrument will extend. The only difficulty or inconvenience attends the application of various apertures to the object-end of large telescopes without disturbing the instrument. This, when the telescope is a refractor of considerable focal length, requires some special contrivance, which, however, is of a very simple character. If near the extremity of the dew-cap a small ledge or shoulder be made in the inside, the apertures corresponding to the magnitudes beyond the power of the finder may be easily raised to their place by a hook of suitable shape at the end of a light wand of sufficient length; and thus the telescope may remain unmoved from the object.

"The same effect which it is proposed to produce by the application of various apertures before the object-glass might be attained by the use of very small eye-holes; precisely the same phenomena occurring as when the aperture of the telescope is diminished at the object-end. It is obvious that when the eye-hole is less than the emergent pencil, its effect is to cut off part of the rays transmitted by the object-glass; and if the eye-hole be duly proportioned to the magnifying power employed, any required aperture may be taken into use and the rest excluded. But though this is a neat and handy mode of diminishing the aperture, there are two grave objections to its use. The first is the difficulty of making eye-holes of precisely the proper size for bringing into use a definite portion of the object-glass; and the second and still more serious objection is, that the same eye-hole will cut off different portions of the object-glass with different magnifying powers; and, consequently, it would always be necessary to employ the same magnifying power, or else to have various sets of eye-holes adapted to the powers usually employed. And thus a complication and difficulty would arise sufficient in most cases to prevent the use of the method altogether.

"I may here mention a plan by which I have succeeded in measuring pretty accurately the diameter of the pupil of my own eye when exposed to no other light than that of the stars. I am not aware that this has hitherto been done. An aperture of about 0·2 of an inch has been assigned to the pupil as probably near the truth on an average; though it no doubt varies considerably in different individuals: but I believe this is not founded on experiment, which indeed may seem to be rendered impossible by the absence of sufficient light. The method I employed is this. Being very short-sighted (requiring the use of Dollond's No. 9 concave), the image of a bright star on the retina of my eye is a luminous disk of large diameter. If a small hole in a card be held close to the eye, the diameter of this luminous disk will be diminished when

1851MNRAS..11..187D

the hole is smaller than the pupil; but when the hole is precisely of the same size as the pupil, the whole of the disk will be included, provided the hole and the pupil are concentric; but the slightest movement of the hole in any direction will cut off a portion of the luminous disk on the opposite side, and change the circular disk into a gibbous one. With a little care the experiment may be made with great nicety; a difference of one or two hundredths of an inch in the size of the hole being immediately perceptible in the effect. By this means I have ascertained the pupil of my eye by starlight to be, as nearly as possible, one quarter of an inch. It is obvious that a person who is not short-sighted may try the same experiment by the use of a convex lens.

"It has occurred to me in the course of these experiments, that diminishing apertures applied to the naked eye might be employed in the determination of star-magnitudes higher than the sixth. It is evident that the scale I have adopted for the lower magnitudes will not apply to these. Calculated by the same formula, the aperture with which a star of the second magnitude should be visible is only the hundredth of an inch. This is much too small: yet with this aperture, *Sirius* is steadily visible, and appears with a planetary disk as if seen through a telescope with a high power in proportion to the aperture. On viewing the sun, and also the full moon, through this extremely small hole, a curious phenomenon presents itself. The edges of their disks are softly defined, and are surrounded by a well-formed bright ring, precisely similar to the first luminous ring round the disk of a bright star viewed with a fine telescope and high power. This is still seen when the hole is as large as the fortieth of an inch in diameter; and this agrees with the telescopic fact, that the emergent pencil must be reduced to about the same size before the ring round a star is separated from its disk. Of course, a moderately dark glass will be required for observing this phenomenon on the sun; and in all cases the edges of the hole should be clean and smooth.

"It might be supposed that in the determination of magnitudes by the use of various apertures on a telescope, the results would be perceptibly affected by the magnifying power employed. But experiment does not support this notion. The mere visibility of stars, even of the faintest, is but slightly affected by the power employed, provided always that the air is pure and the moon absent, or not within 40° or 50° of the object. Indeed, it is a fact that an increase of power brings out many small stars which were invisible with a lower magnifier. But this has its limits; and there is a certain ratio of illuminating and magnifying power which is both more pleasant to the eye and more efficient than any other. This I believe to be attained when the diameter of the emergent pencil is about 0·02 of an inch. Consequently, the power $16\frac{1}{2}$ on my finder is too great to be the most efficient for the apertures below 0·3 of an inch; and when the aperture is reduced to 0·15 of an inch, and the diameter of the emergent pencil is therefore only 0·009 of an inch, the appearance of a star of the sixth magnitude,

1851MNRAS..11..187D

to which that aperture corresponds, resembles that of a faint planetary nebula. But this is quite consistent with the object being steadily seen when the eye rests upon it; just as a dull planet, such as *Saturn*, viewed with a telescope in full sunshine, may perhaps not be seen at all till the eye has for some time searched over the field of view; yet, after being caught sight of, it remains steadily visible when the eye is directed fully upon it. Generally, however, a lower power, as 8 or 10, might be more agreeable and less fatiguing to the eye when the aperture employed is less than 0·3 of an inch. And this may be especially desirable in determining the standard aperture for stars of the sixth magnitude, from which the apertures suitable to the inferior magnitudes are to be deduced.

"It is interesting and desirable to ascertain the magnitude of the smallest star visible in a telescope of any given size and construction; and this may readily be deduced from the formula. For since $\frac{\alpha}{a} = 2^{\mu - m}$, the value of μ for any given values of α and a may be found. Adapting the formula to logarithmic computation, we have $\mu = \frac{\log \alpha - \log a}{\log 2} + m$. If, for example, it be desired to know precisely what magnitude should be attributed to the *minimum visibile* in my $8\frac{1}{2}$-foot refractor, whose aperture is 6·34 inches; where, as in my own case, $a = 0{\cdot}15$ when $m = 6$, we have $\mu = \frac{0{\cdot}8021 - 9{\cdot}1761}{0{\cdot}3010} + 6 = 11{\cdot}34$, the magnitude sought, according to the Struvian scale. And since $H = 2\,\Sigma - 6$, the corresponding Herschelian magnitude $= 16{\cdot}68$. In like manner the *minimum visibile* in the finder of my equatoreal $= 9{\cdot}35\ \Sigma = 12{\cdot}70\ H$. And a star only visible by fits, or steadily visible only on extraordinarily brilliant nights, may be safely denominated of $9\frac{1}{2}$ magnitude.

"It is obvious that any numerical magnitude and its corresponding aperture suited to the ocular sensibility of any observer, may be made the standard; or may represent m and a respectively. And as the management of very small apertures on a telescope armed with a comparatively high power, is less easy than that of the larger apertures with the same power, it may be found by some observers more satisfactory to fix upon another magnitude (the 8th, for instance), as the standard, and to compute the numbers of their table of apertures from the aperture suitable to their eye for that magnitude. The formula I have given, being general, applies equally to this mode of proceeding; but for higher magnitudes than the standard value of m, whatever that may be assumed, the exponent of the power of 2 will be negative; the numerical value of μ being less than that of m. For this purpose, a list of stars whose magnitudes have been carefully determined, and conveniently situated for comparison at different times of the year, may form a desirable appendix to this communication, which I hope at some future time to do myself the pleasure of presenting to the Society."